Suspension and Granular Media: Wet vs. Dry

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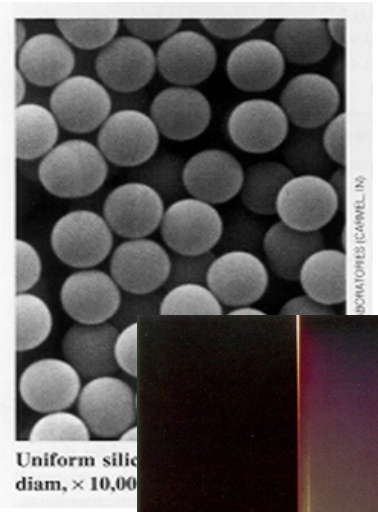
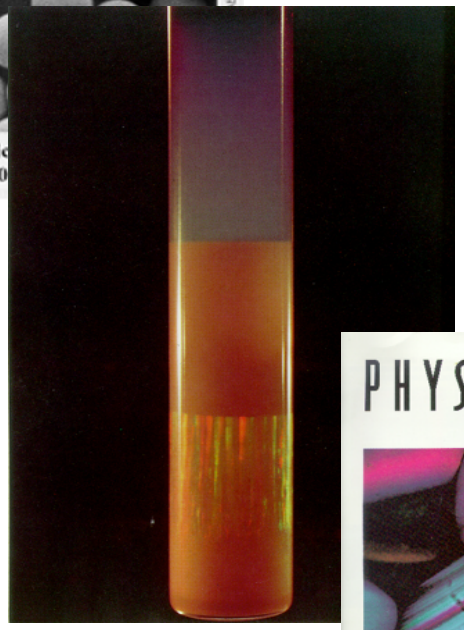
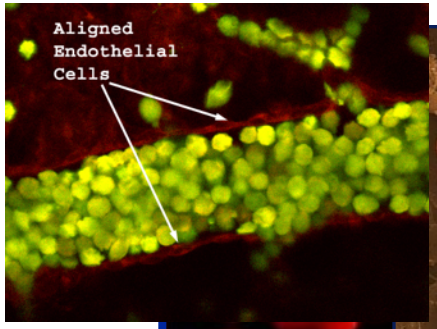
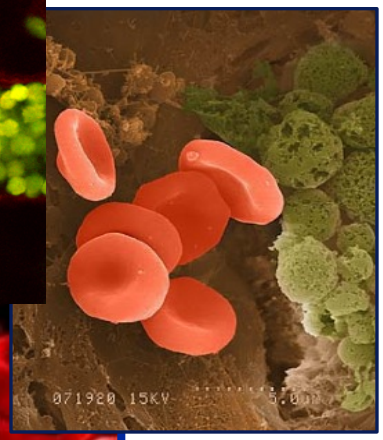
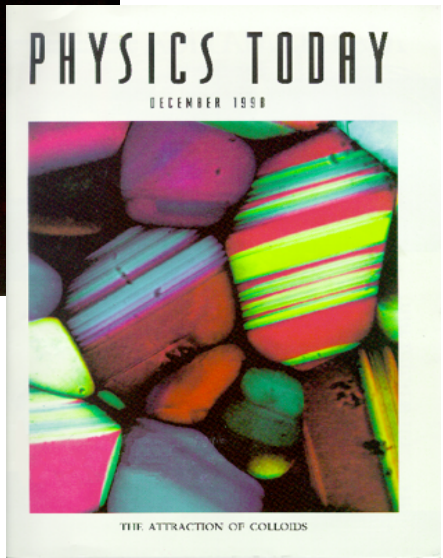
Suspensions



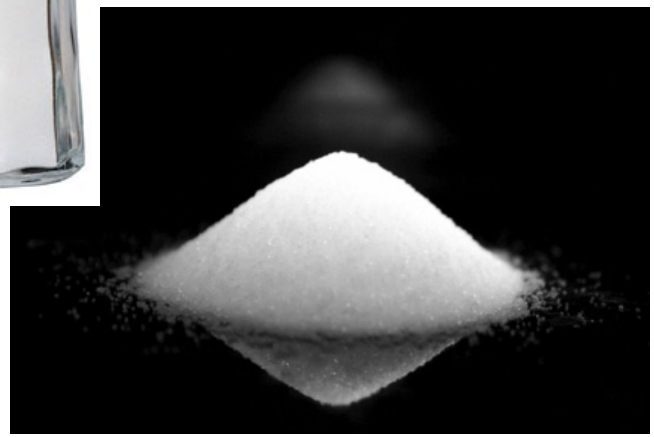


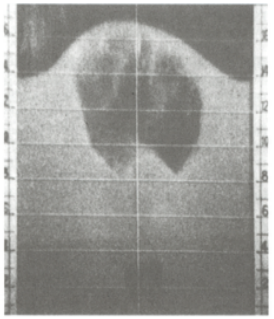


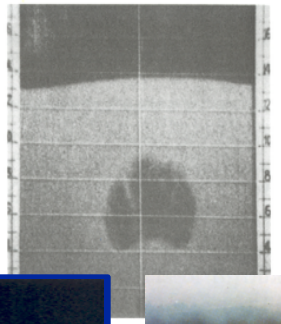


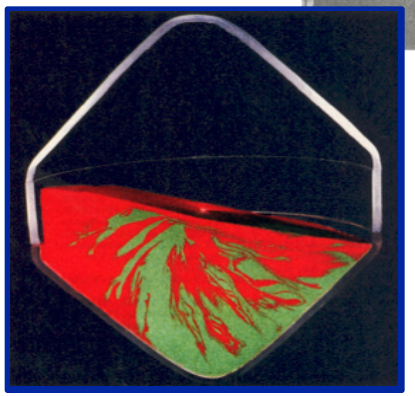


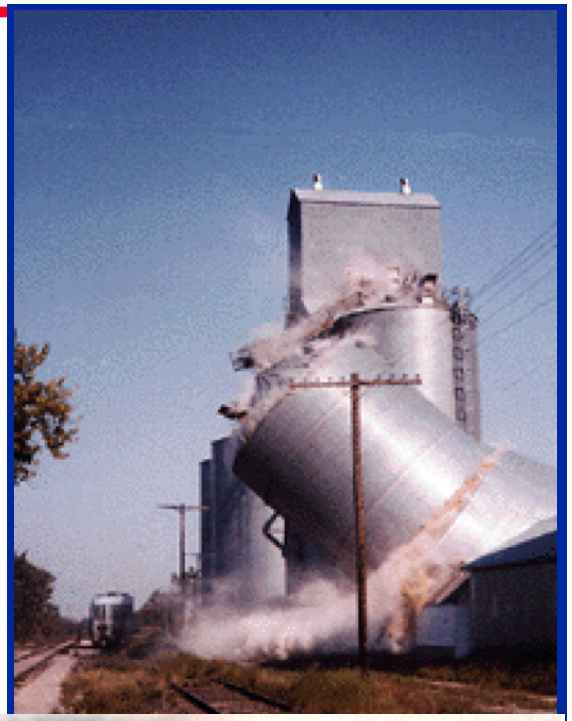
Granular Media







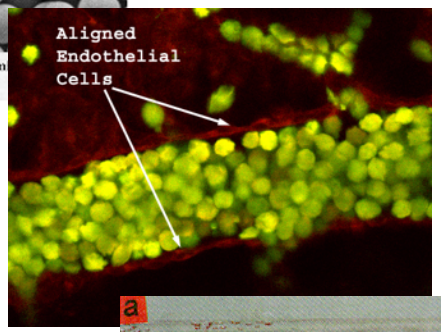




Suspensions vs. Granular Media

Particles dispersed in a liquid

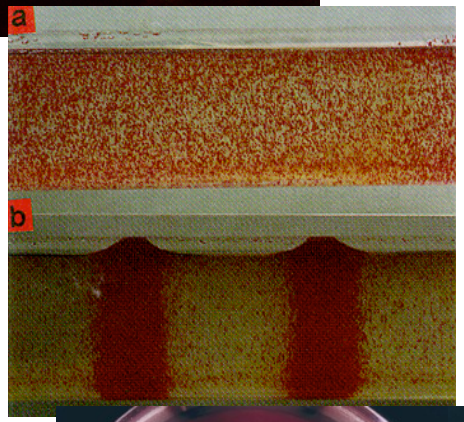
Particles dispersed in a gas



Wet

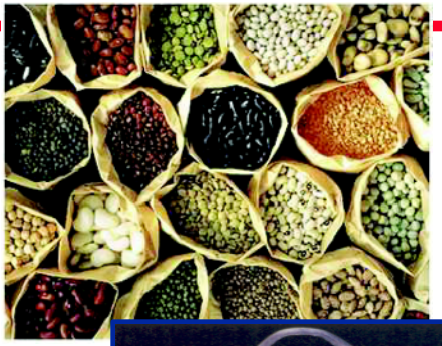
10nm

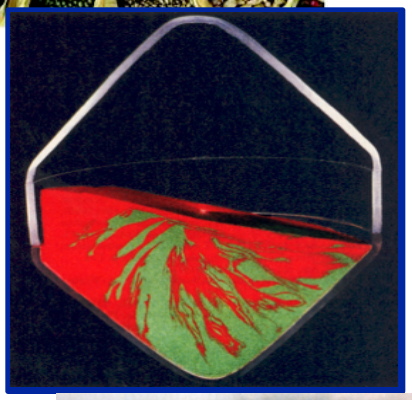
1mm

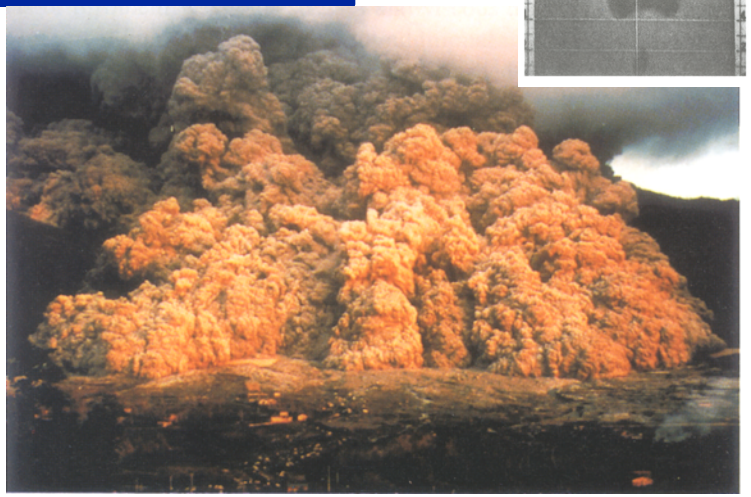


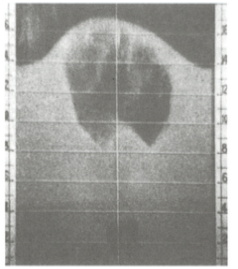
?

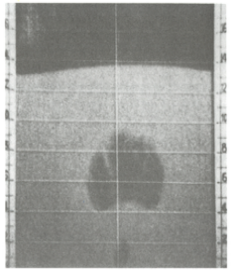
Dry







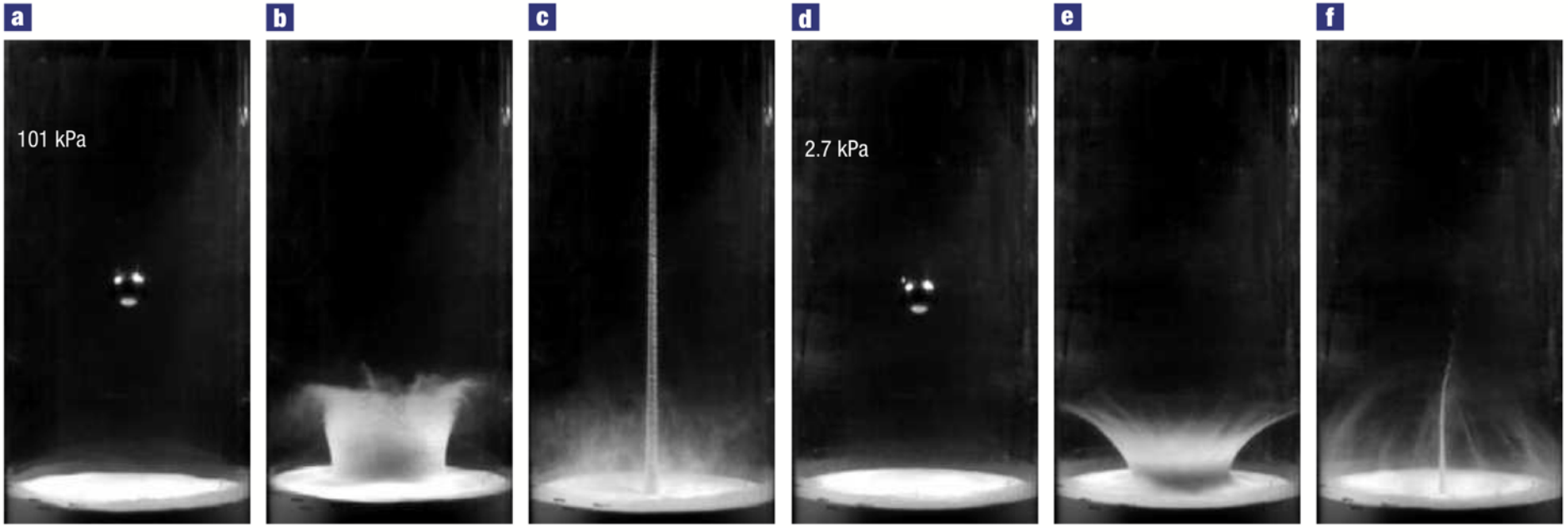




1mm

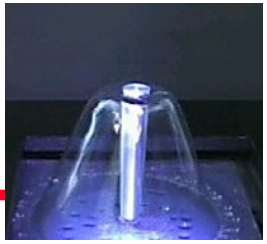
10m

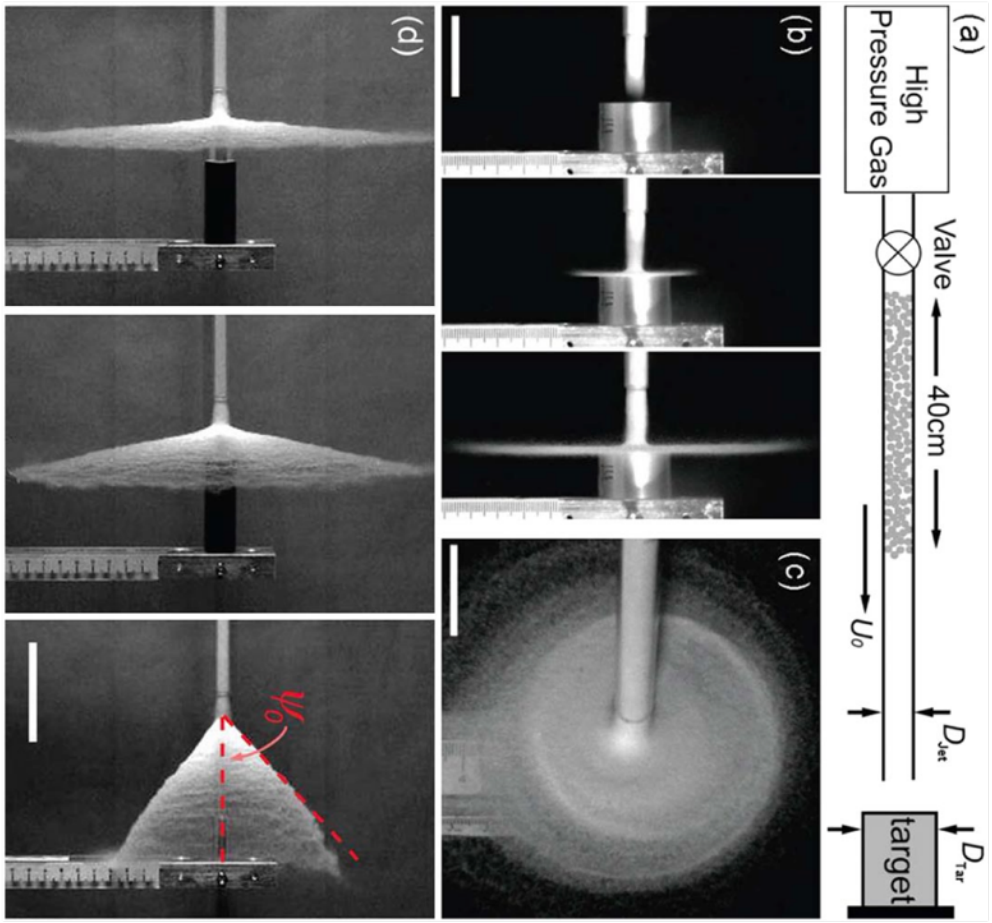
Granular Splash



Royer *et al* (2005)

Granular Jet/Water Bells





Cheng *et al* (2007)

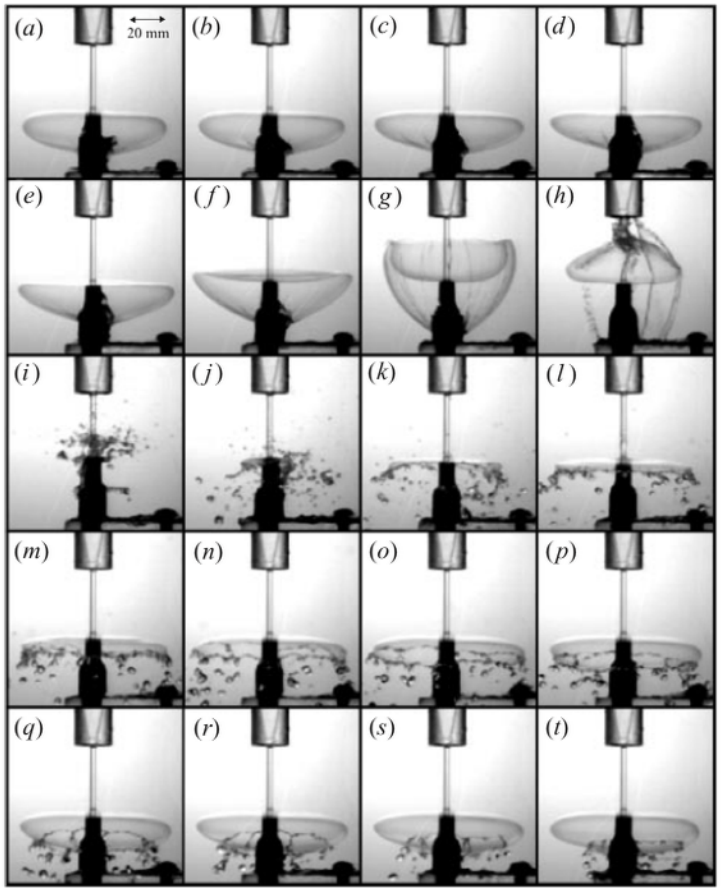
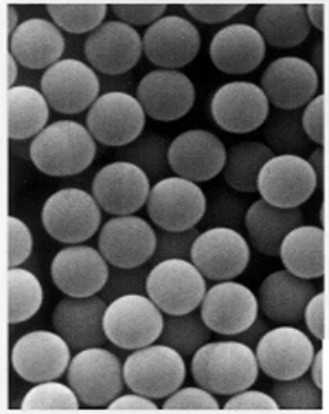


FIGURE 17. Instability of a closed water bell characterized by a large angle of ejection $\psi_0 = 87°$, observed with $D_0 = 3.0\,\text{mm}$, $D_i = 10.0\,\text{mm}$ and $U_0 = 2.21\,\text{m s}^{-1}$. Time increases from (*a*) to (*t*) with the time step $\Delta t = 27.7\,\text{ms}$.

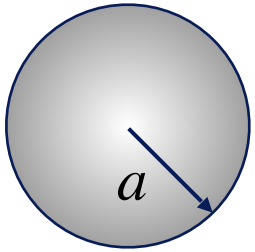
Clanet (2001)

Overly (Very) Simplified Model



Uniform silica microspheres (1 μm diam, × 10,000).

Particle Dynamics



$$m\frac{du}{dt} = -kx - \zeta u + F^{ext}$$

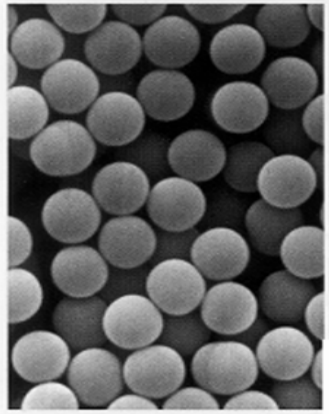
A 'representative' particle

Particle inertia

Interactive force with neighbors

'Viscous' dissipation

Overly (Very) Simplified Model



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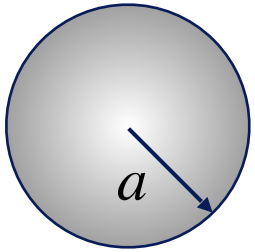
Particle Dynamics



$$m\frac{du}{dt} = -kx - \cancel{\zeta}u + F^{ext}$$

$$\eta a$$

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Particle inertia

Interactive force with neighbors

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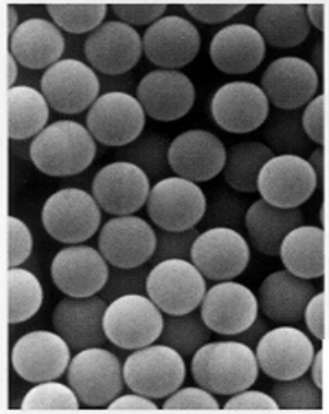
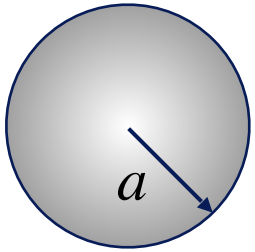
Overly (Very) Simplified Model


Uniform silica microspheres (1 μm diam, × 10,000).

Particle Dynamics



$$m\frac{du}{dt} = -kx - \zeta u + F^{ext}, \qquad \zeta = \eta a$$

A 'representative' particle

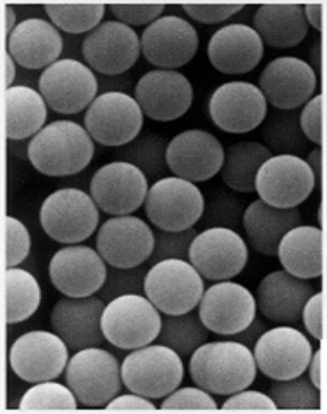
Particle inertia

Interactive force with neighbors

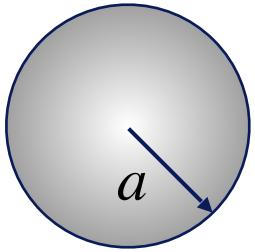
'Viscous' dissipation

External Forcing

Overly (Very) Simplified Model


Uniform silica microspheres (1 μm diam, × 10,000).

Particle Dynamics



$$m\frac{du}{dt} = -kx - \zeta u + F^{ext}$$

$$\zeta = \eta a$$

A 'representative' particle

Particle inertia

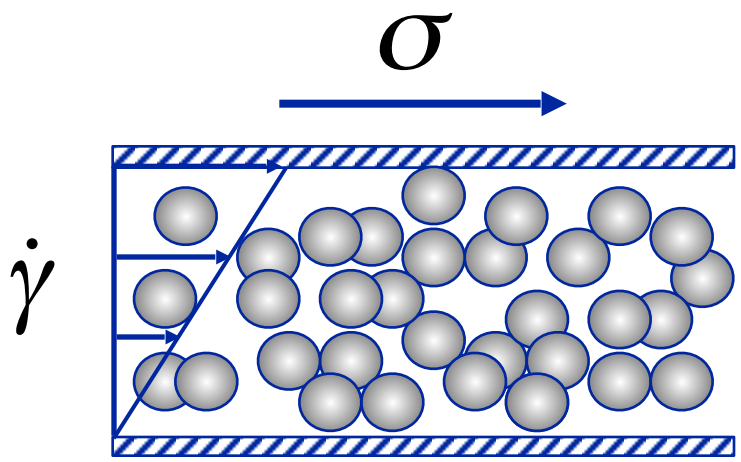
Interactive force with neighbors

'Viscous' dissipation

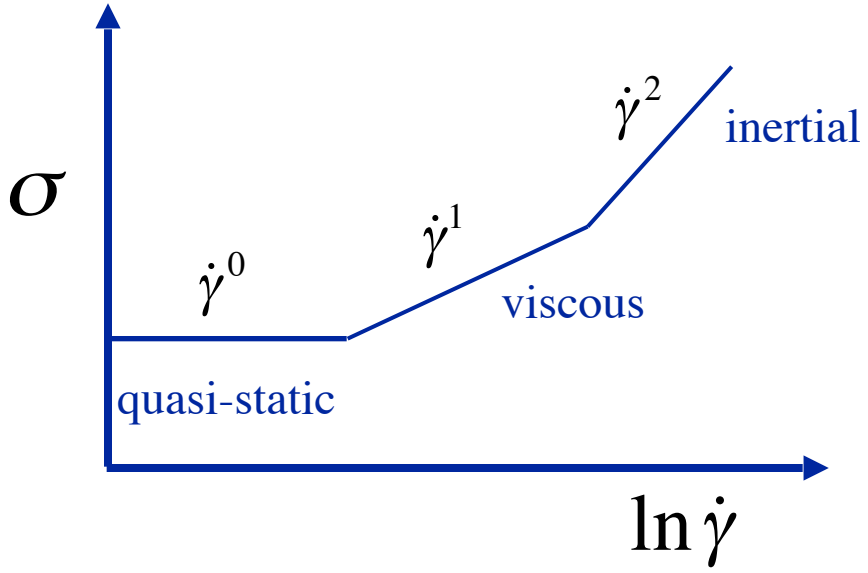
External Forcing

We can anticipate (at least) three different behaviors depending on what balances the external forcing.

Common Point: Flow Behavior or Rheology?



Stress: $-n\langle xF \rangle$



Particle Dynamics

$$m\frac{du}{dt} = -kx - \underbrace{\zeta}_{\eta a}u + F^{ext}$$

Hooke: $kx \sim F^{ext} \rightarrow \sigma \sim k/a$

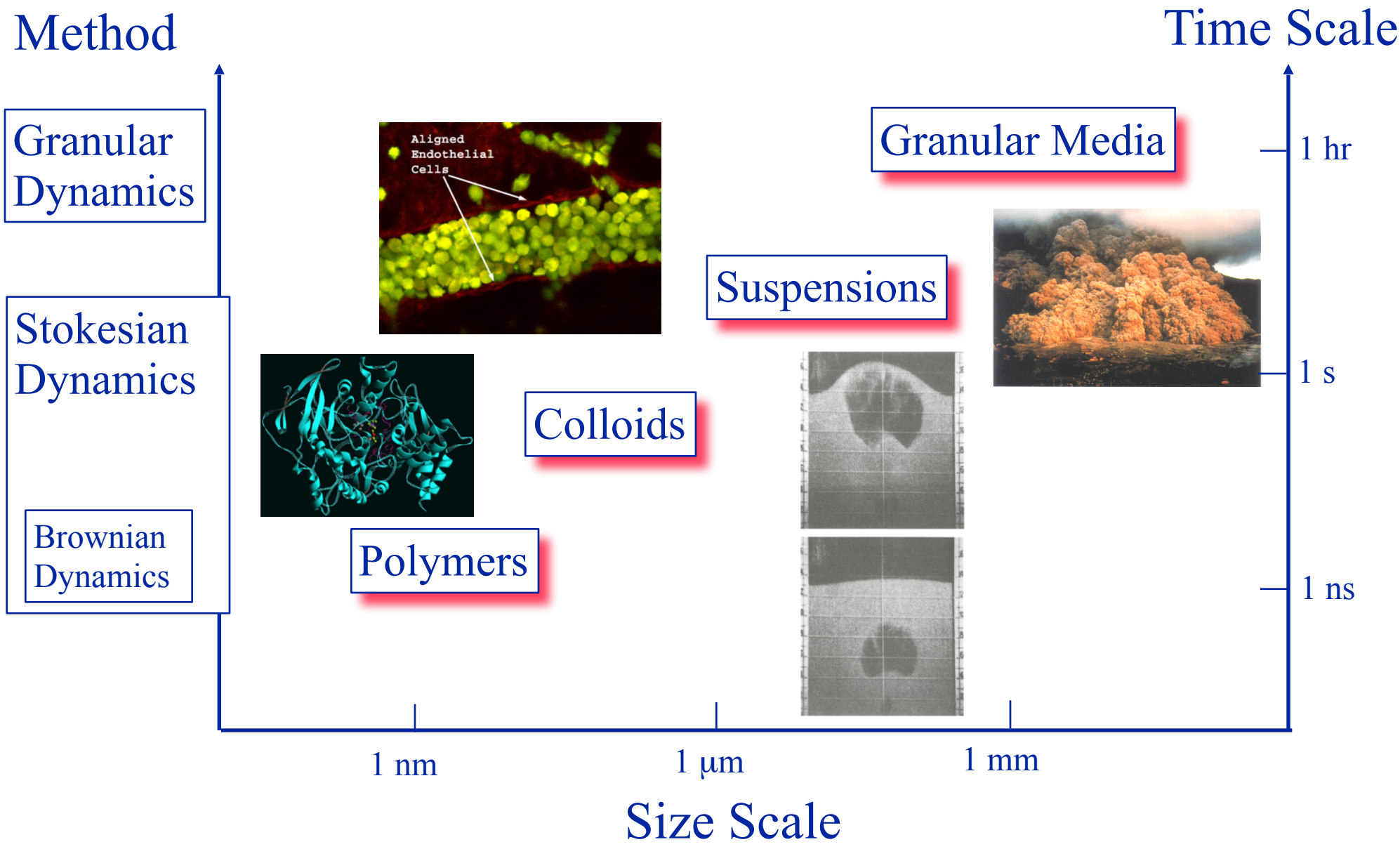
Stokes: $\eta a u \sim F^{ext} \rightarrow \sigma \sim \eta\dot{\gamma}$

Newton: (Bagnold) $m\frac{du}{dt} \sim F^{ext} \rightarrow \sigma \sim \rho_p a^2 \dot{\gamma}^2$

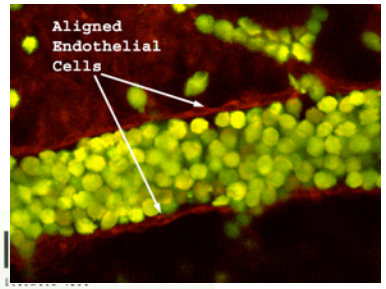
Outline

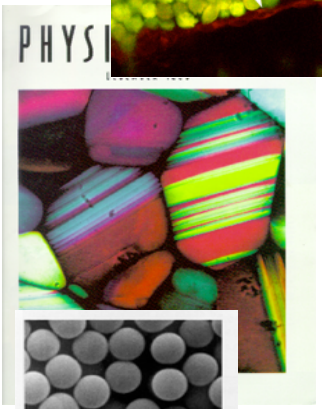
- Explore this possible connection between wet and dry.
- Discuss the 'micromechanics' – the underlying physics at the particle level.
- 'Rapid' flows of these materials – not yielding or static behavior, nor motion induced by vibrations.
- Discuss rheology, diffusion and phase segregation.
- Wet means fully wet – not 'moist' granular media. Liquid bridges give rise to attractive forces and modify the behavior substantially.
- Stable dispersions – those composed of hard, repulsive particles.

Length and Time Scales of Particulate Material



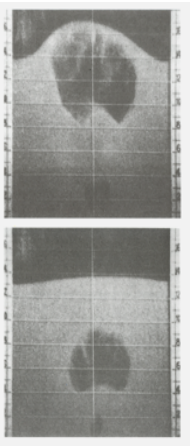
Two-phase particulate matter





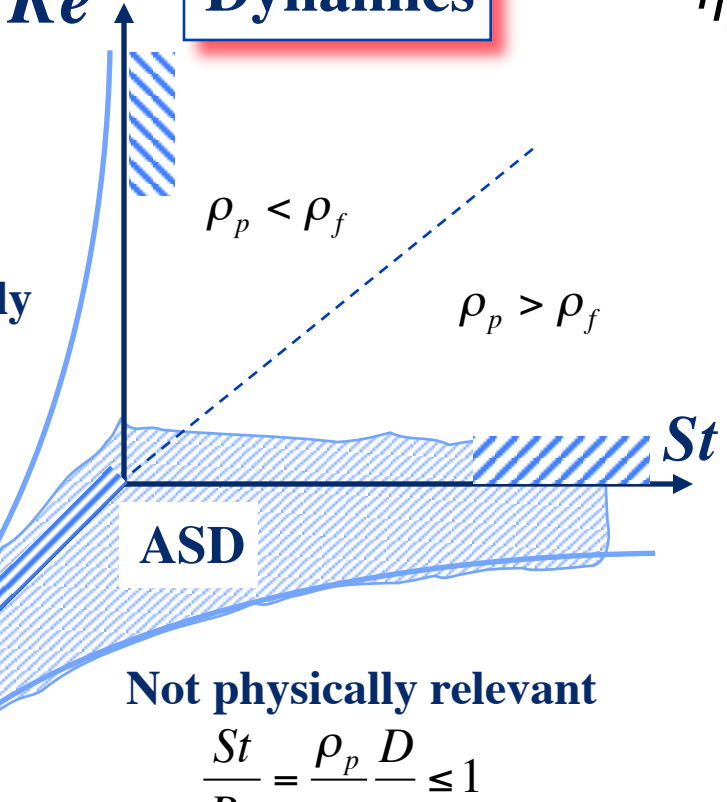
Bubble Dynamics

$$Re = \frac{\rho_f a^2 \dot{\gamma}}{\eta_f}$$



$$St = \frac{\rho_p a^2 \dot{\gamma}}{\eta_f}$$

Granular Dynamics



Not physically relevant

$$\frac{Re}{Pe} = \frac{D}{\nu} << 1$$

Not physically relevant

$$\frac{St}{Pe} = \frac{\rho_p}{\rho_f}\frac{D}{\nu} \leq 1$$

$$Pe = \frac{a^2 \dot{\gamma}}{D}$$

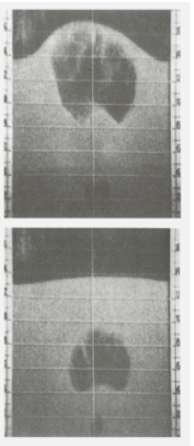
Stokesian Dynamics

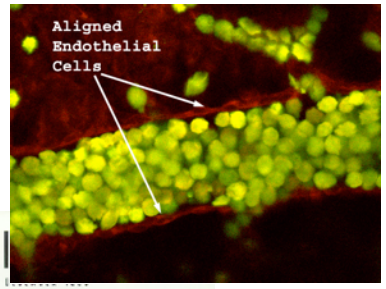
Two-phase particu

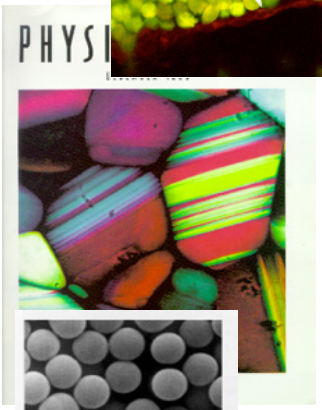
Yurkovetsky & Brady *POF* (1995)
"Statistical Mechanics of Bubbly Liquids"

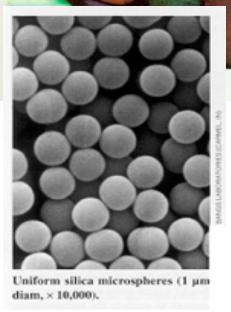
Bubble Dynamics

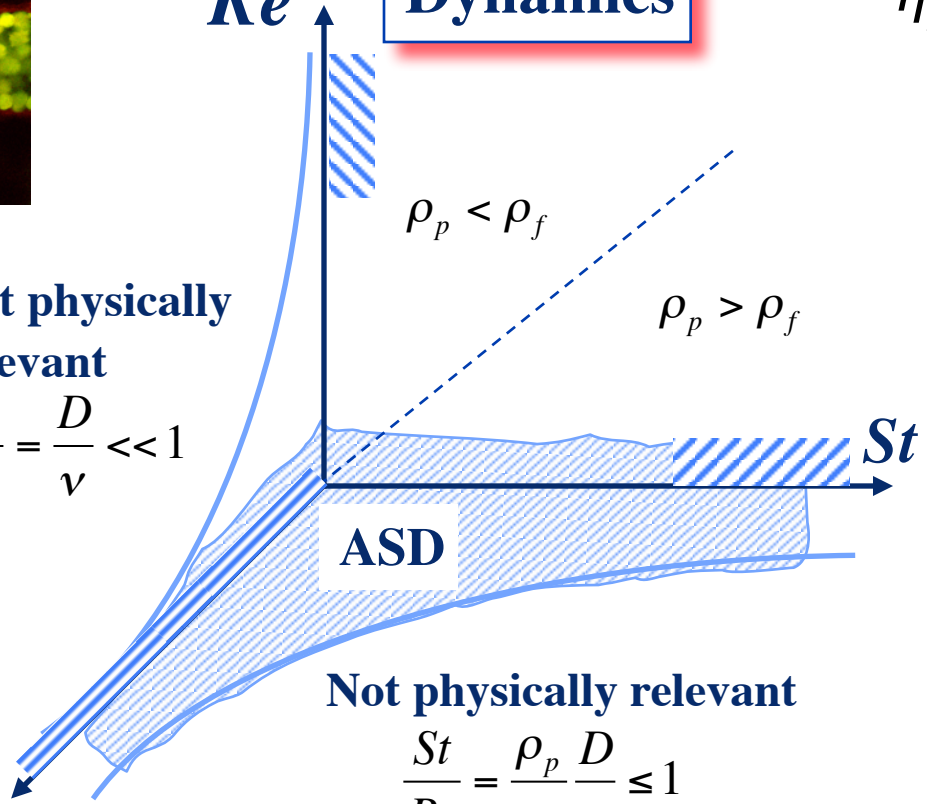
$$Re = \frac{\rho_f a^2 \dot{\gamma}}{\eta_f}$$











Not physically relevant

$$\frac{Re}{Pe} = \frac{D}{\nu} << 1$$

$$St = \frac{\rho_p a^2 \dot{\gamma}}{\eta_f}$$

Granular Dynamics

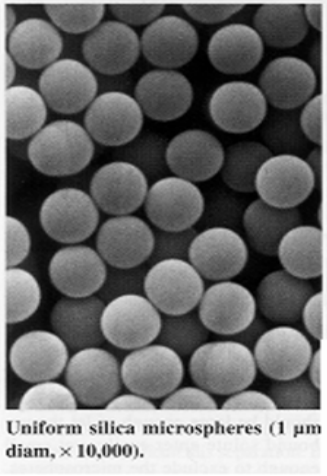
Not physically relevant

$$\frac{St}{Pe} = \frac{\rho_p}{\rho_f}\frac{D}{\nu} \leq 1$$

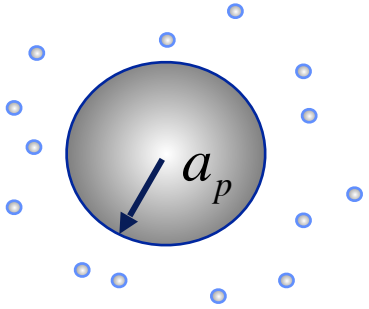
$$Pe = \frac{a^2 \dot{\gamma}}{D}$$

Stokesian Dynamics

Micromechanics



Continuum Approximation: $a_p >> a_s$



$$N_s \sim \left(a_p / a_s\right)^3 N_p$$

$$\tau_s \sim a_s / \sqrt{3kT/m} \approx 10^{-13}\, s$$

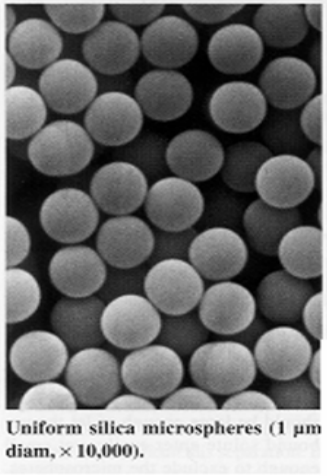
$$\tau_s \sim a_s^{\,2} / \nu \;\; , \; \nu = \eta / \rho$$

$$\tau_p / \tau_s \sim \left(a_p / a_s\right)^2$$

Therefore, the solvent can be treated as a continuum :

$$\rho \frac{D\boldsymbol{u}}{Dt} = -\nabla p + \eta \nabla^2 \boldsymbol{u} \;\; , \;\; \nabla \cdot \boldsymbol{u} = 0$$

Micromechanics



Uniform silica microspheres (1 μm diam, × 10,000).

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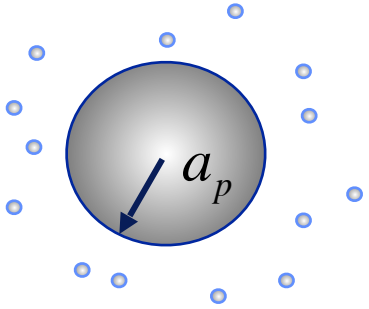


$$N_s \sim \left(a_p / a_s\right)^3 N_p$$

$$\tau_s \sim a_s / \sqrt{3kT/m} \approx 10^{-13}\, s$$

$$\tau_s \sim a_s^{\,2} / \nu \;\;,\; \nu = \eta / \rho$$

$$\tau_p / \tau_s \sim \left(a_p / a_s\right)^2$$

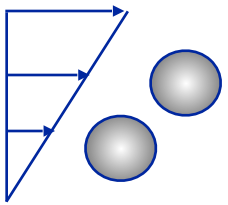
Therefore, the solvent can be treated as a continuum :

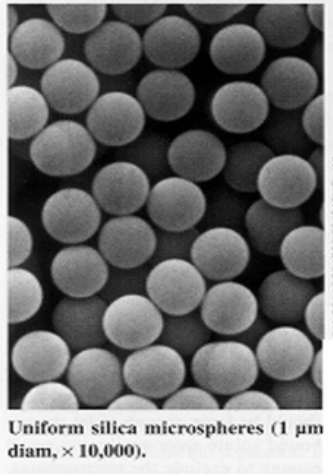
$$Re = \frac{\rho U a}{\eta} << 1$$

$$\cancel{\rho \frac{D\boldsymbol{u}}{Dt}} = -\nabla p + \eta \nabla^2 \boldsymbol{u} \;\;,\;\; \nabla \cdot \boldsymbol{u} = 0$$

Micromechanics: Particle Inertia ($St \neq 0$)

Particle Motion: $$m \cdot \frac{dU}{dt} = F^H + \cancel{F^B} + F^P$$





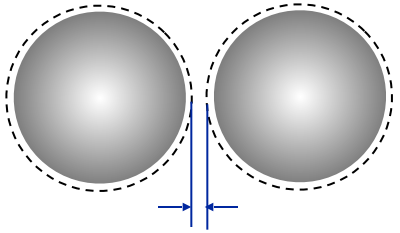
Uniform silica microspheres (1 μm diam, × 10,000).

Hydrodynamic: $$F^H = -R(x) \cdot \left(U - U^\infty\right)$$

Stokes drag

Dissipation: e , η

Interparicle/ external: $F^P =$ Elastic/inelastic collisions:



$\delta = 10^{-3}a$

$$St = \tau_p \dot{\gamma} = m\dot{\gamma} \Big/ 6\pi\eta a \sim \rho_p \dot{\gamma} a^2 \Big/ \eta$$

$$Re = \frac{\rho_f}{\rho_p} St << 1$$

Viscosity Across Scales

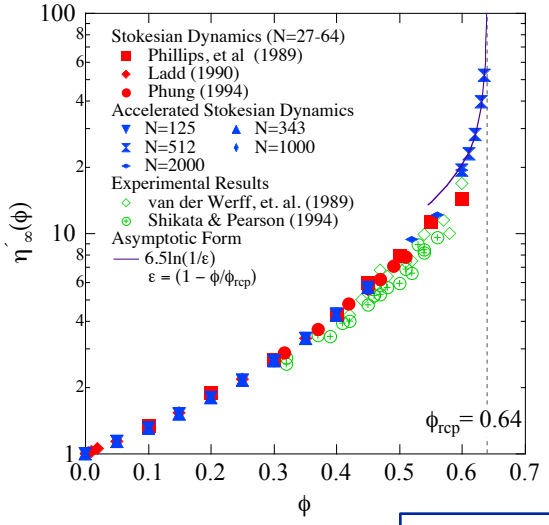
Method

Time Scale

Granular Dynamics

Stokesian Dynamics

Brownian Dynamics



$Pe \leq O(1)$

Colloids

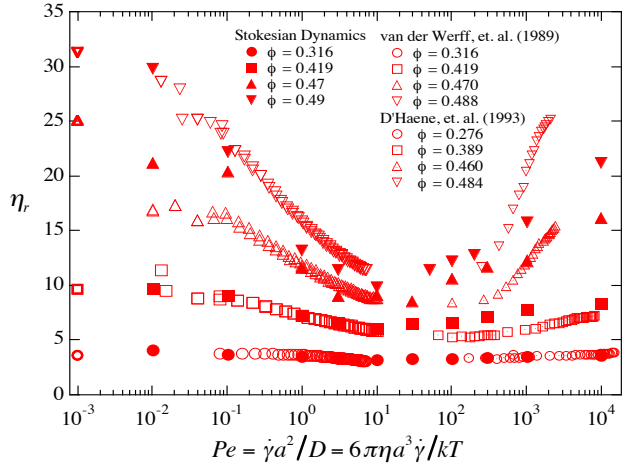
Polymers

$Pe >> 1 \ , \ St \neq 0$

Granular Media

Suspensions

$Pe >> 1 \ , \ St = 0$



1 hr

1 s

1 ns

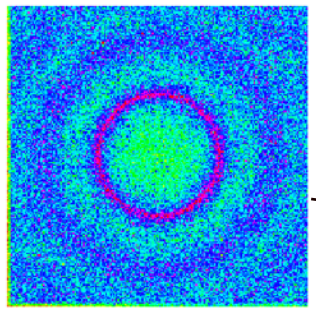
1 nm

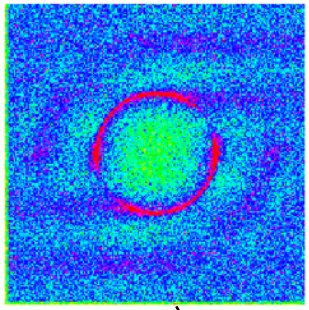
1 μm

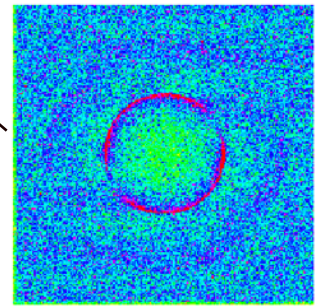
1 mm

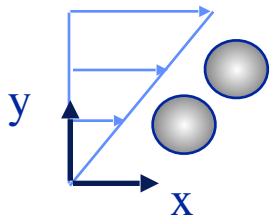
Size Scale

Brownian & hydrodynamic contributions to stress









Stokesian Dynamics

$\phi = 0.45$

Total

Hydrodynamic

Brownian

η_r

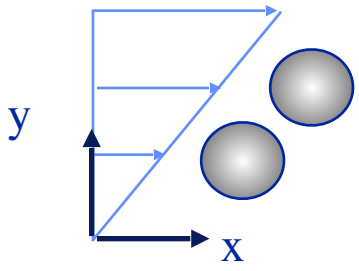
$\eta_r^B(\phi)$

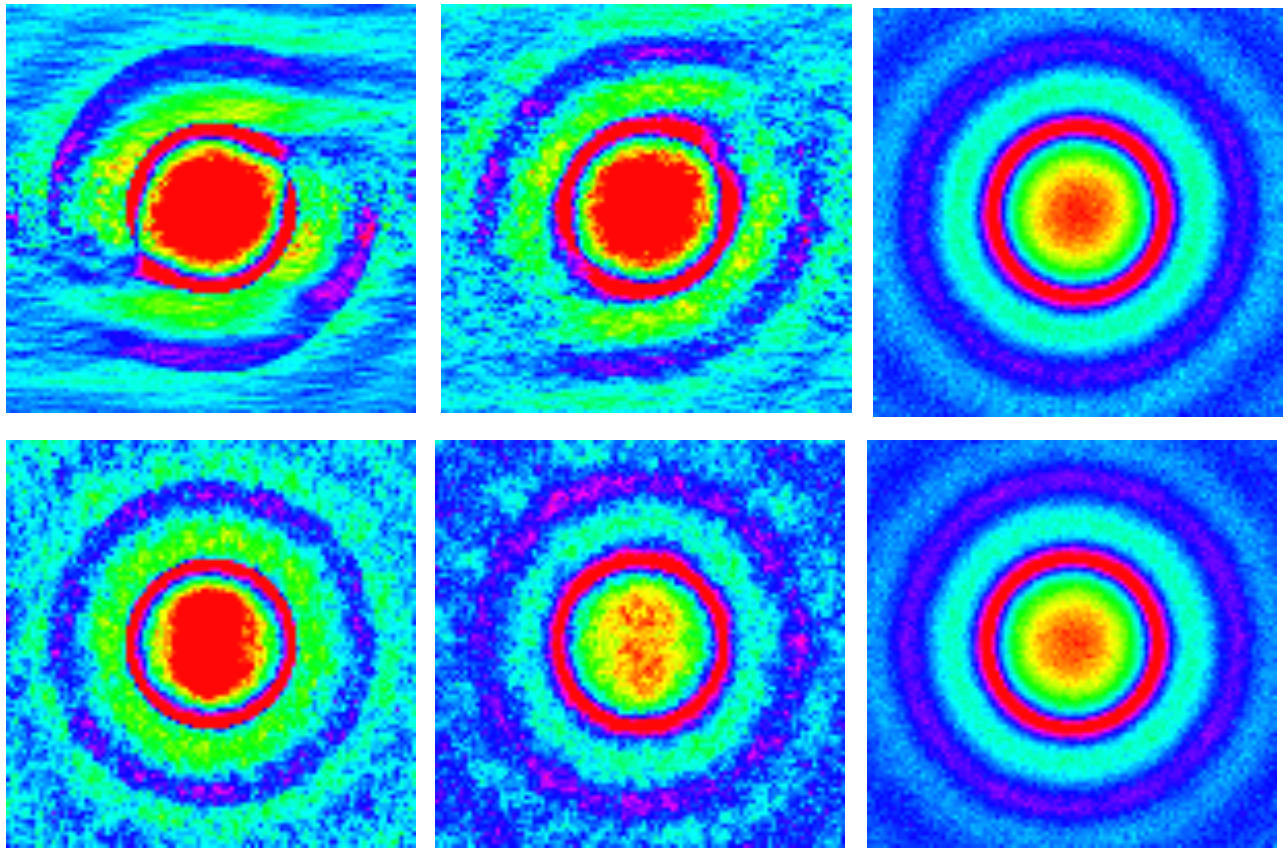
$\eta_r^H(\phi)$

$Pe = \dot{\gamma} a^2 / D = 6\pi\eta a^3 \dot{\gamma} / kT$

Microstructure Evolution with St, $\phi = 0.40$

$St \sim \rho_p \dot{\gamma} a^2 / \eta$





g_{xy}

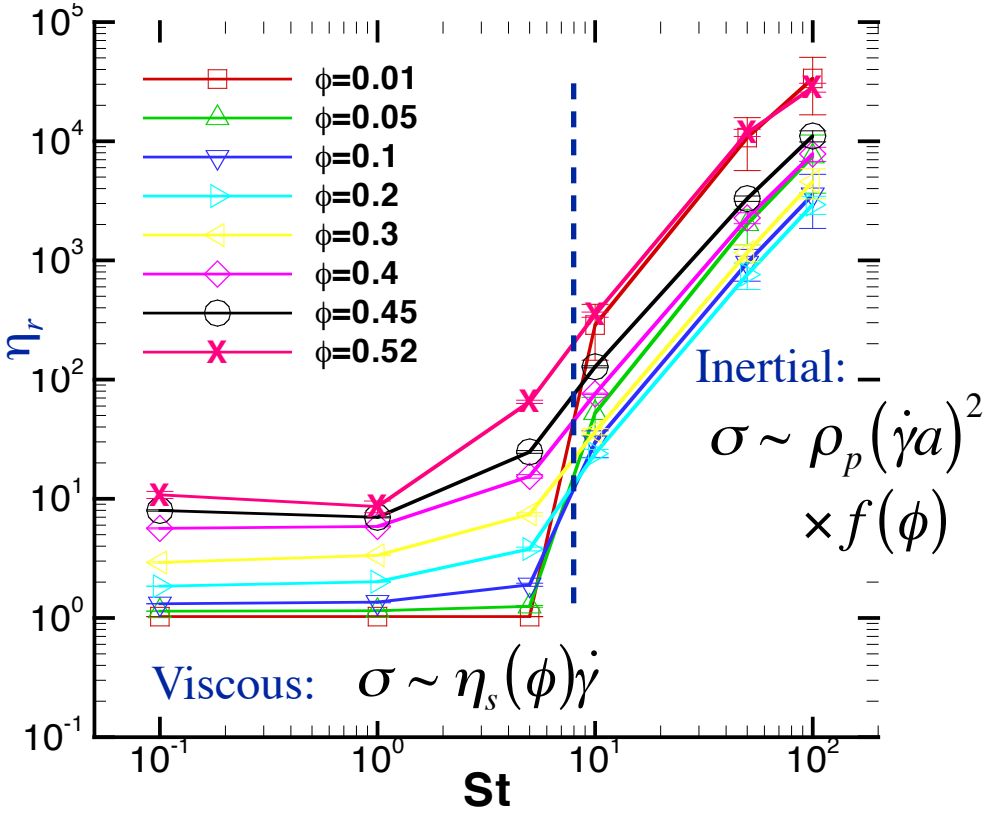
g_{yz}

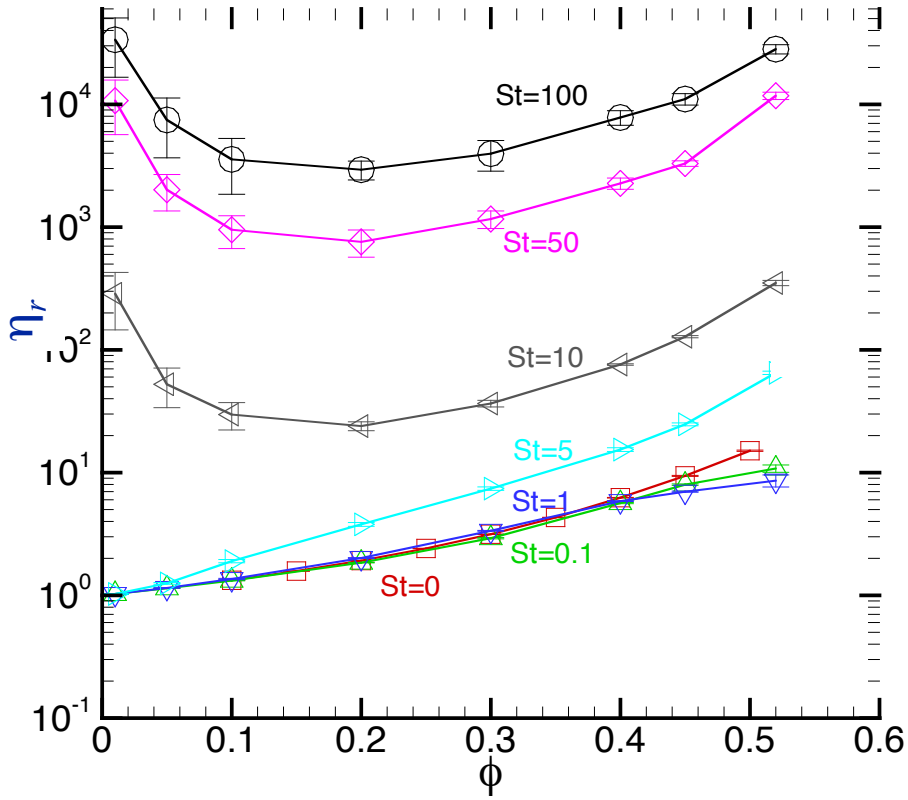
$St = 0.1$ $St = 10$ $St = 100$

Structure becomes more isotropic with increasing particle inertia

Shear viscosity vs ϕ and *St*

$$St \sim \rho_p \dot{\gamma} a^2 / \eta$$

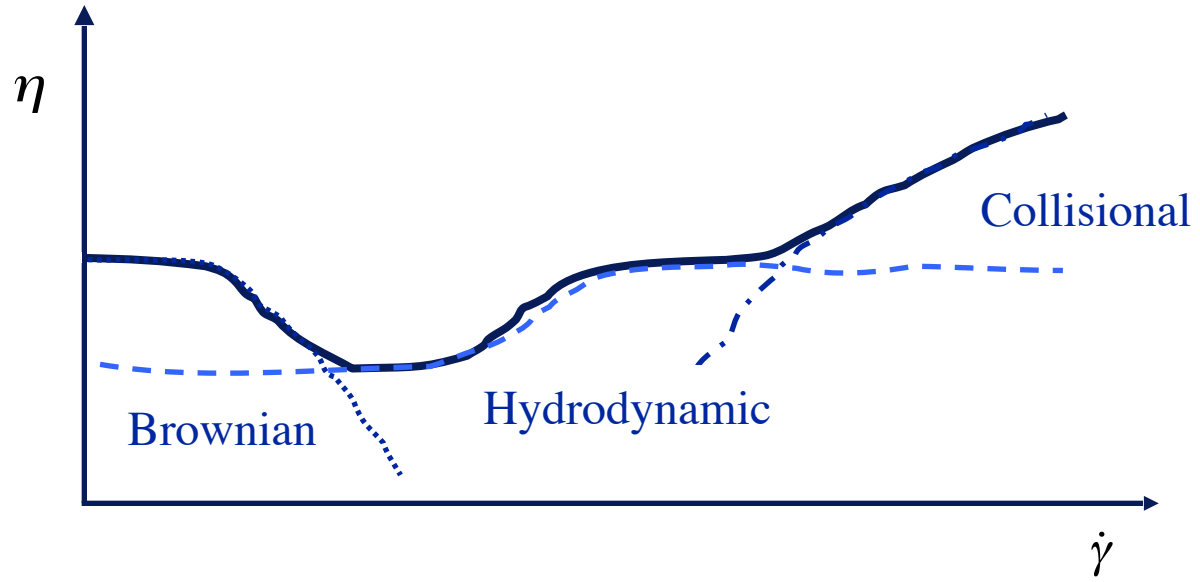




Stress can be modeled as a sum of viscous and inertial contributions

Remarks I

- The total stress is approximately the sum of the Brownian, Hydrodynamic and Collisional terms.



- There are also normal stresses that have been determined and are very important for suspension flow modeling.

- All stresses increase with volume fraction, dramatically so near maximum packing.

Diffusion Across Scales

Method

Granular Dynamics

Stokesian Dynamics

Brownian Dynamics

Time Scale

$Pe >> 1\ ,\ St \neq 0$

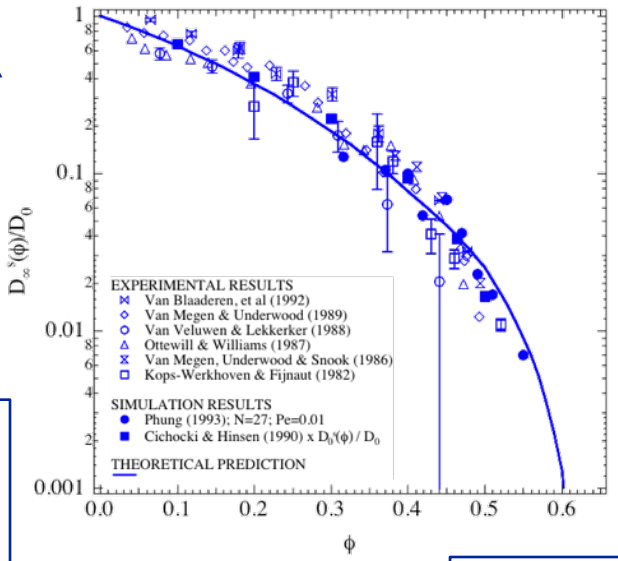
Granular Media

1 hr

Suspensions

$Pe >> 1\ ,\ St = 0$

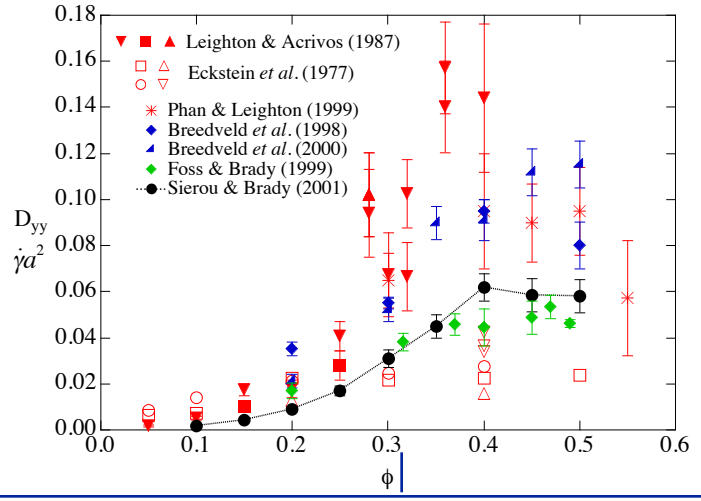
1 s



$Pe \leq O(1)$

Colloids

Polymers



1 ns

1 nm

1 μm

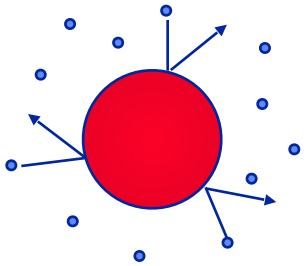
1 mm

Size Scale

Particle Diffusion: $D \sim (v')^2 \times \tau$

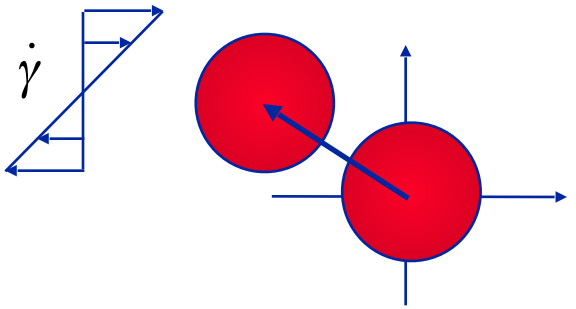
- Brownian motion (Stokes-Einstein-Sutherland)

$$(v')^2 \sim \frac{3kT}{m}, \quad \tau \sim \frac{m}{6\pi\eta a}, \quad D \sim \frac{kT}{2\pi\eta a}$$



- Shear-induced

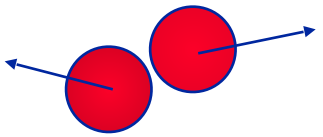
$$(v')^2 \sim (\dot{\gamma}a)^2, \quad \tau \sim \dot{\gamma}^{-1}, \quad D \sim \dot{\gamma}a^2$$



- Granular Gas (Kinetic Theory)

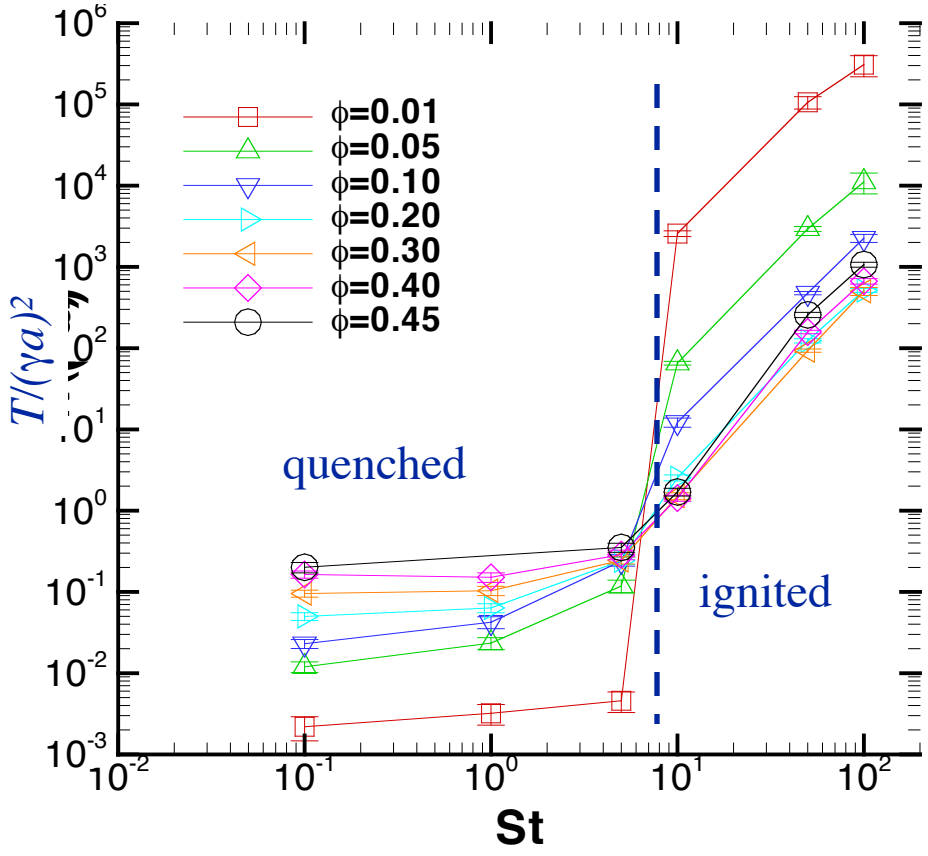
$$(v')^2 \sim T_G, \quad \tau \sim \frac{\lambda}{\sqrt{(v')^2}}, \quad D \sim \lambda\sqrt{T_G} \sim \dot{\gamma}a\lambda$$

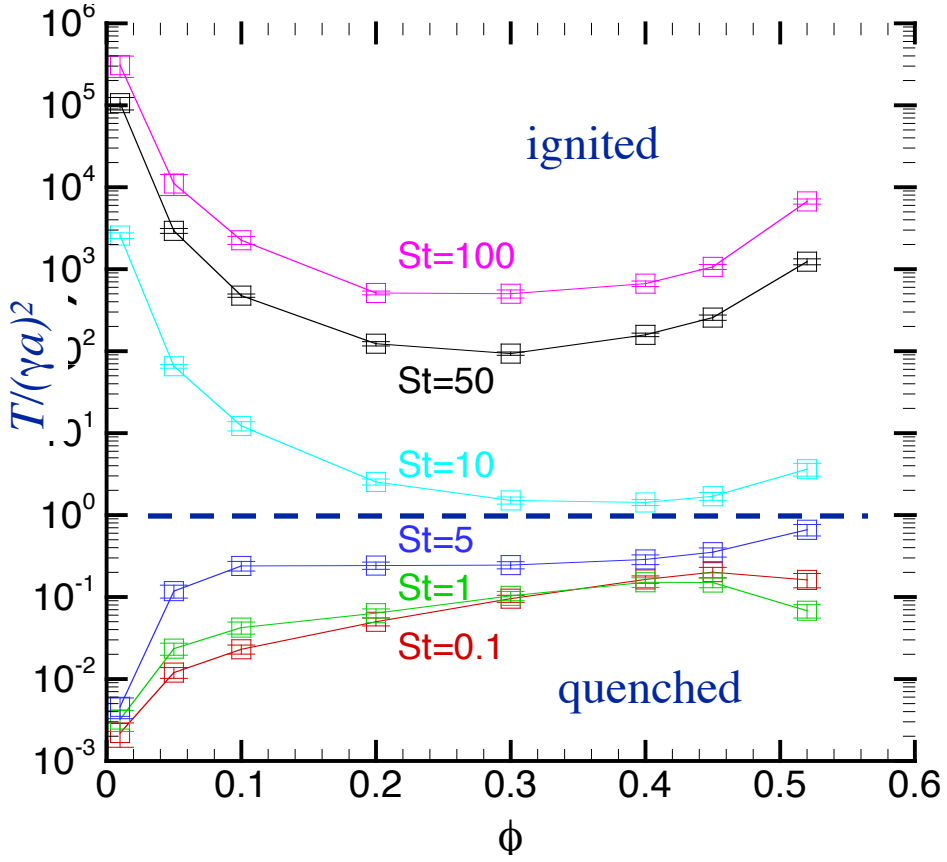
$$T_G \sim (\dot{\gamma}a)^2$$



‘Suspension temperature’ vs *St* and ϕ

$$St \sim \rho_p \dot{\gamma} a^2 / \eta$$



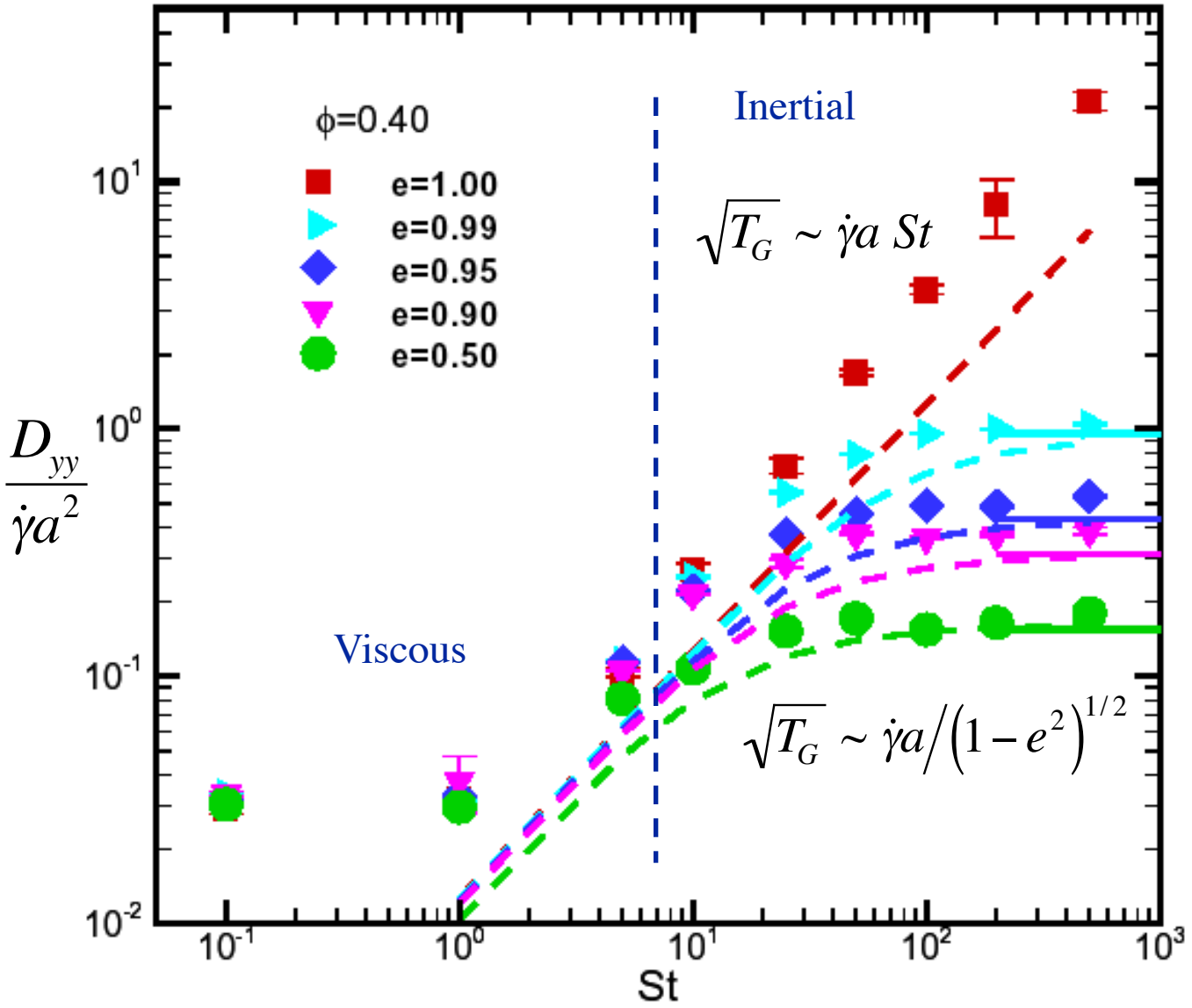


Two regimes: a ‘quenched’ (low *T*) and an ‘ignited’ (high *T*) state

Self Diffusion vs. *St*

Savage & Dai (1993):

$$D = \frac{a\sqrt{\pi}\sqrt{T_G}}{4(1+e)\phi g_0(\phi)}$$



Energy balance:

(Sangani *et al* 1996)

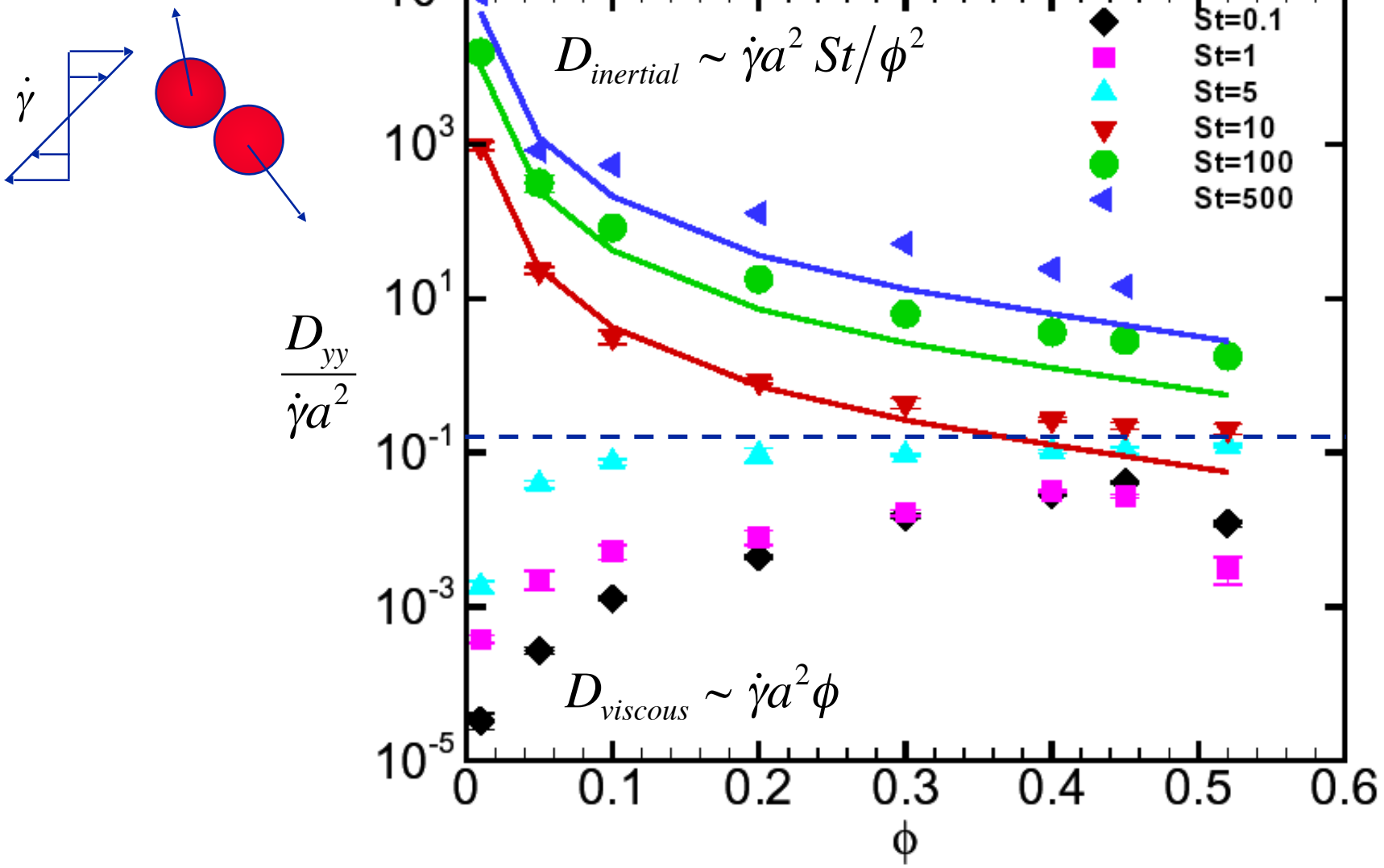
$$\eta_s \dot{\gamma}^2 = \dot{\Phi}_{viscous} + \dot{\Phi}_{inelastic}$$

$$\dot{\Phi}_{viscous} \sim \eta a^{-2} \phi\, T_G$$

$$\dot{\Phi}_{inelastic} \sim \rho_p a^{-1} (1-e^2) \phi^3 T_G^{3/2}$$

$$\eta_s \sim \rho_p a \sqrt{T_G}$$

Self Diffusion at finite *St*



Remarks II

- **Particle Diffusion:**

a, $\dot{\gamma}$ ↓

Brownian :	$D \sim kT/\eta a$	↓	as	ϕ	↑	
Viscous :	$D \sim \dot{\gamma} a^2$	↑	as	ϕ	↑	
Inertial :	$D \sim \lambda\sqrt{T_G}$	↓	as	ϕ	↑	for $St \geq 10$

- **Implications:**

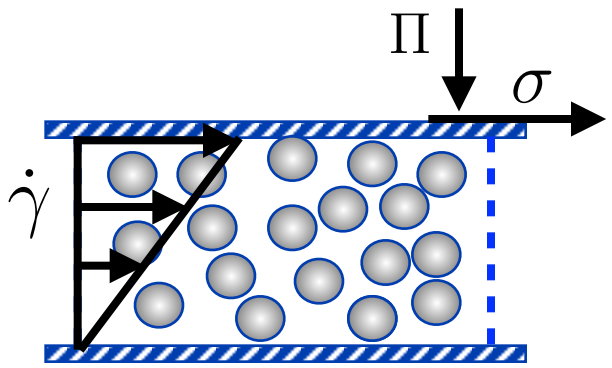
Mesoscopic : $$\frac{\partial g(\boldsymbol{r})}{\partial t} + \dot{\gamma}\,\nabla \cdot \boldsymbol{U}(\boldsymbol{r}) g(\boldsymbol{r}) = \nabla \cdot \boldsymbol{D}(\boldsymbol{r}) \cdot \nabla g(\boldsymbol{r}) \quad ?$$

$$Pe \sim O(1)$$

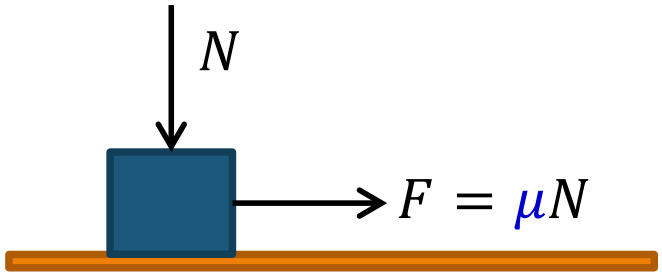
Macroscopic : when $\dot{\gamma}$ varies spatially $\Rightarrow$

- particle migration
- phase separation
- pattern fomration

Dense Suspensions: Friction coefficient μ



Analogy with classical mechanics:



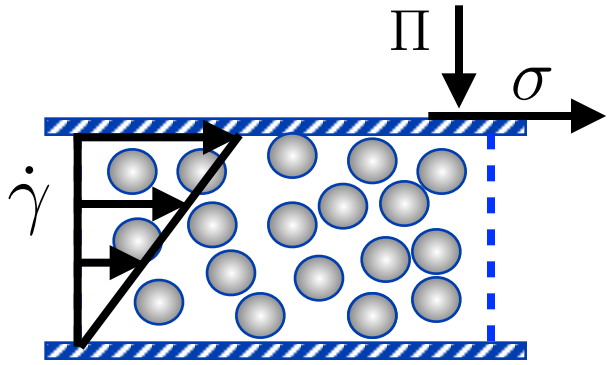
NonBrownian suspensions

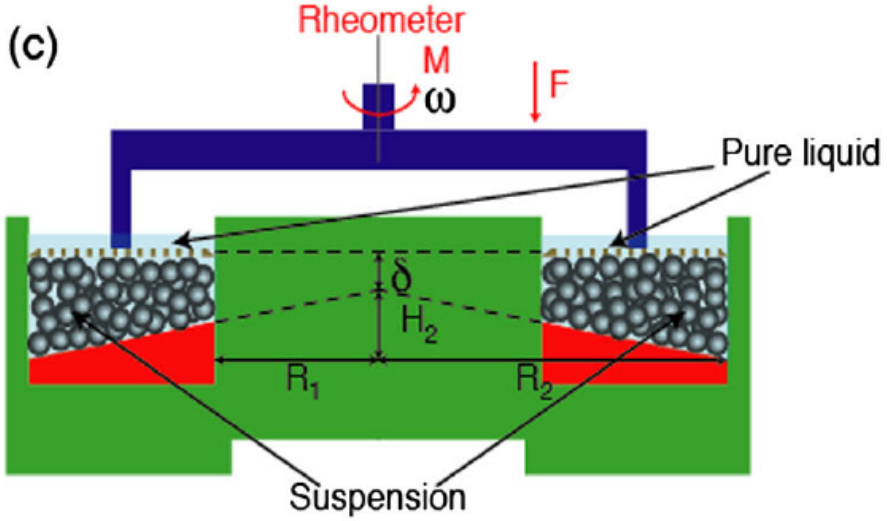
$$\sigma = \eta\dot{\gamma}\,\eta_s(\phi)$$

$$\Pi = \eta\dot{\gamma}\,p(\phi)$$

$$\mu = \frac{\sigma}{\Pi} = \frac{\eta_s(\phi)}{p(\phi)} \sim O(1) \text{ as } \phi \to \phi_{max}$$

Dense Suspensions: Friction coefficient μ





Boyer *et al PRL* (2011)

NonBrownian suspensions

$$\sigma = \eta\dot{\gamma}\,\eta_s(\phi)$$

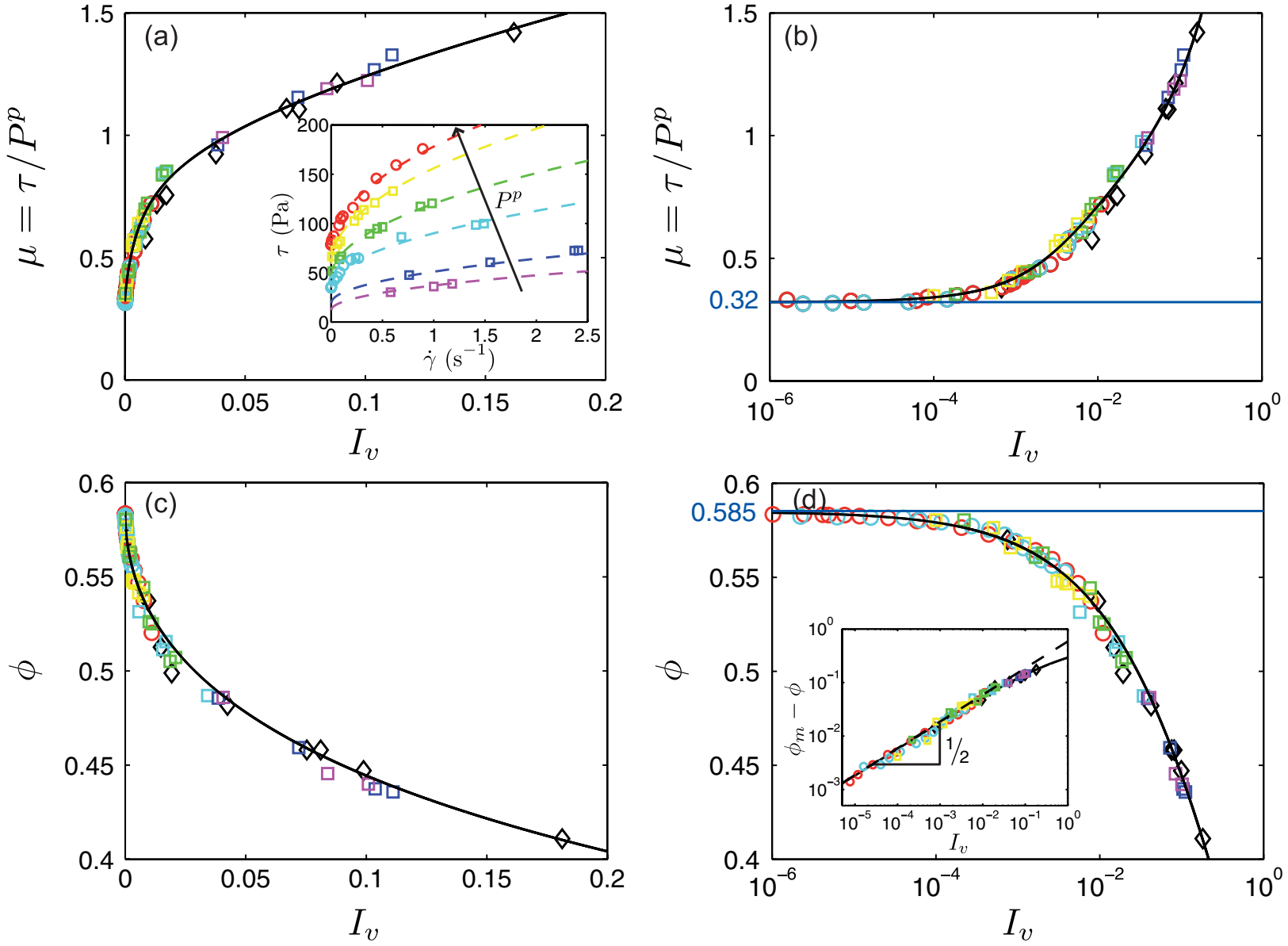
$$\Pi = \eta\dot{\gamma}\,p(\phi)$$

$$\mu = \frac{\sigma}{\Pi} = \frac{\eta_s(\phi)}{p(\phi)} \sim O(1) \text{ as } \phi \to \phi_{max}$$

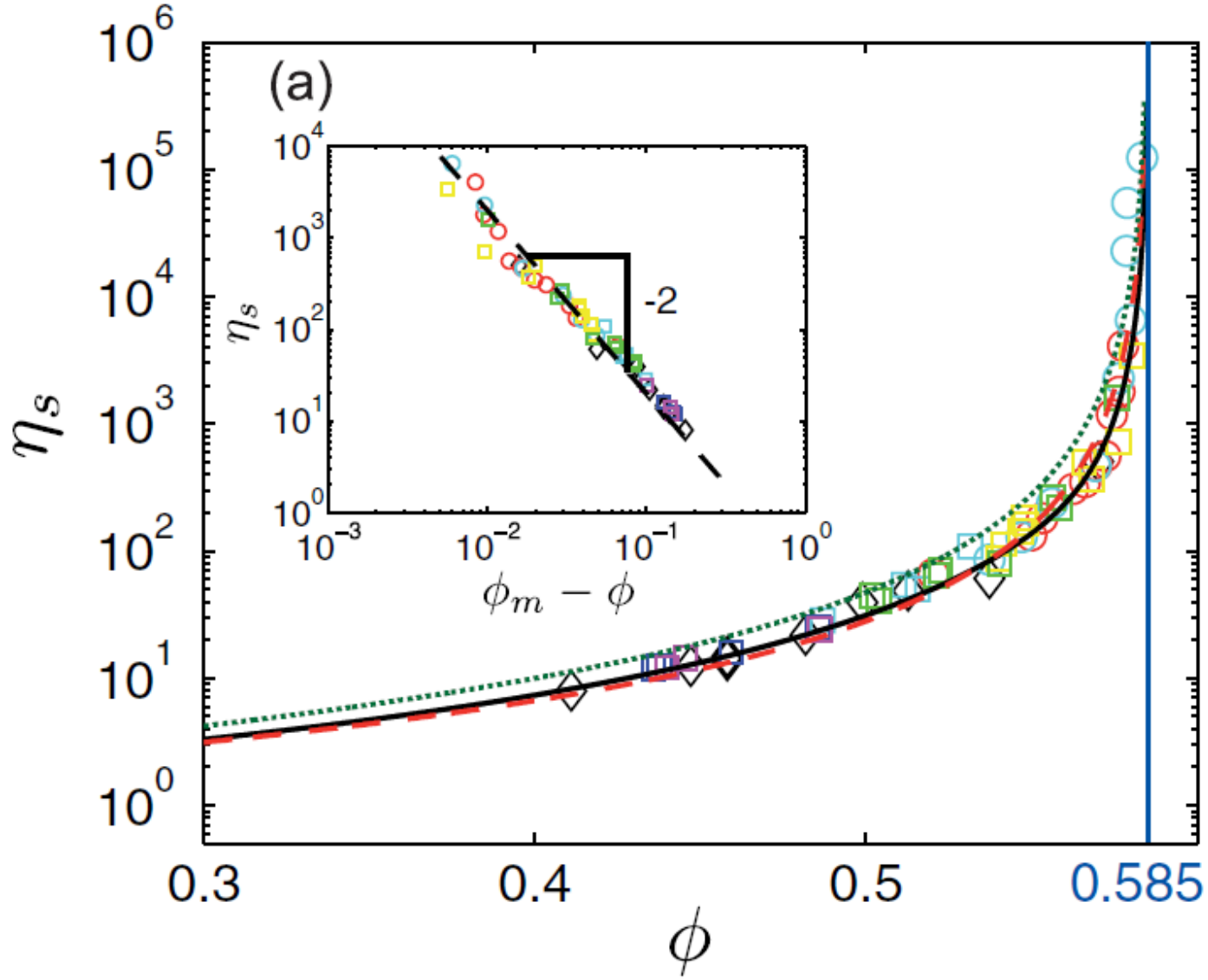
$$I_v = \eta\dot{\gamma}/\Pi = 1/p(\phi)$$

Boyer *et al* (*PRL* 2011)

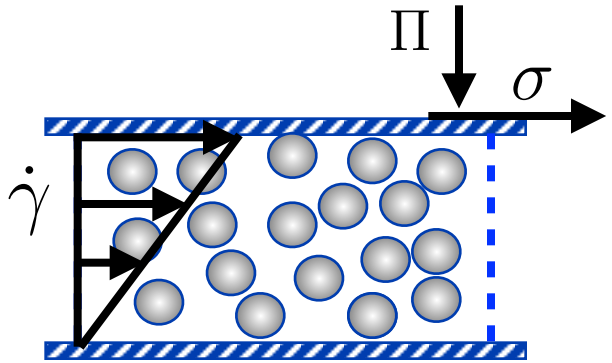
$$I_v = \eta\dot{\gamma}/\Pi = 1/p(\phi)$$



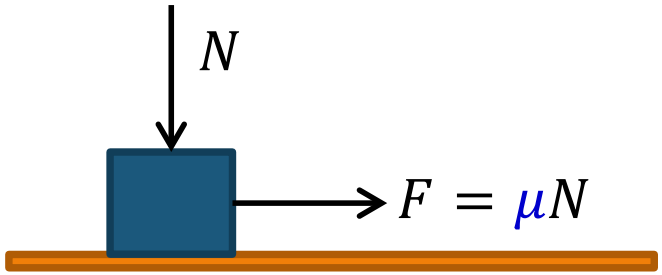
Boyer *et al* (*PRL* 2011): Effective Viscosity



Friction coefficient μ: Granular suspensions



Analogy with classical mechanics:



NonBrownian suspensions

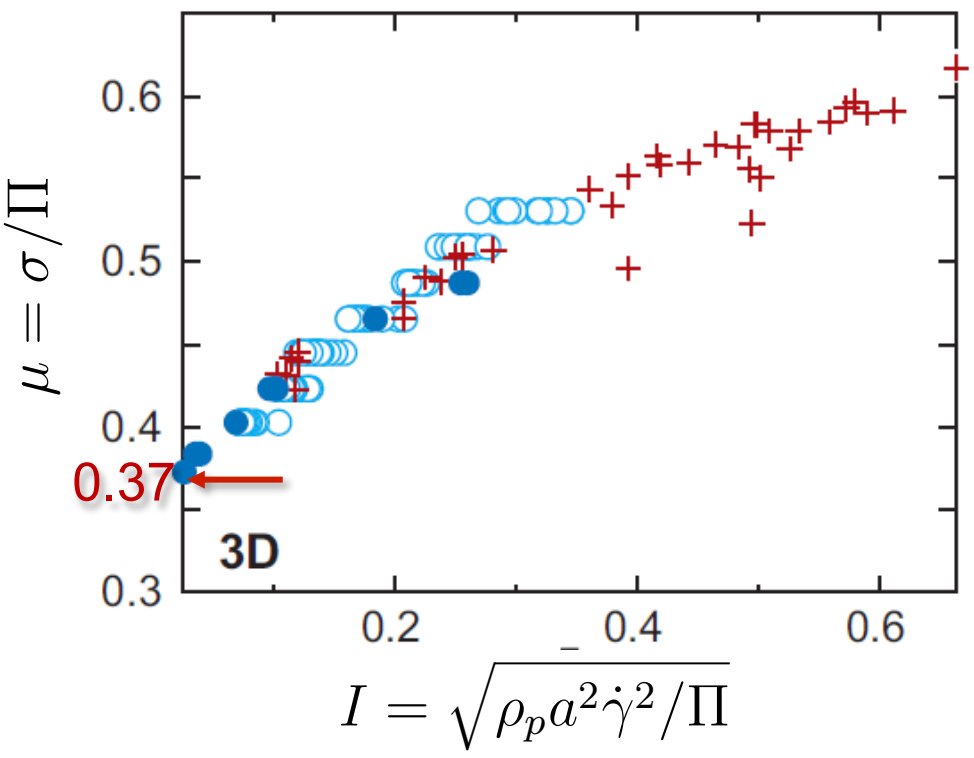
$$\sigma = \eta\dot{\gamma}\,\eta_s(\phi)$$
$$\Pi = \eta\dot{\gamma}\,p(\phi)$$
$$\mu = \frac{\sigma}{\Pi} = \frac{\eta_s(\phi)}{p(\phi)}$$

Granular suspensions

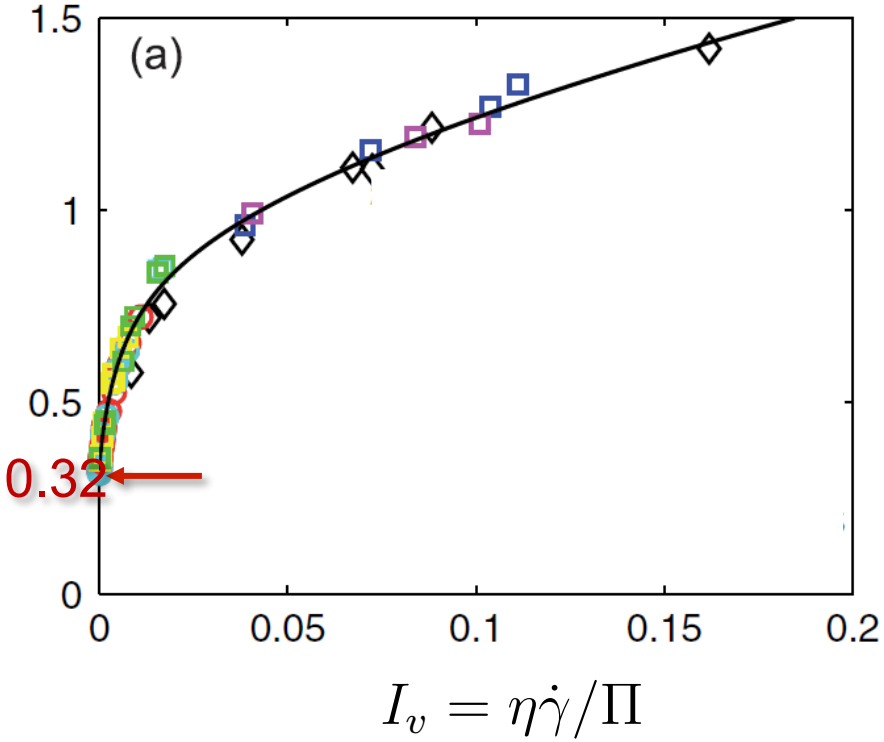
$$\sigma_G = \rho_p\dot{\gamma}^2 a^2\,\eta_G(\phi)$$
$$\Pi_G = \rho_p\dot{\gamma}^2 a^2\,p_G(\phi)$$
$$\mu_G = \frac{\sigma_G}{\Pi_G} = \frac{\eta_G(\phi)}{p_G(\phi)}$$

Granular and wet nonBrownian suspensions

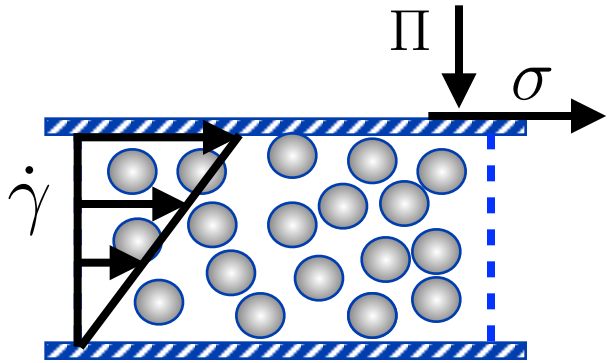
Dry Granular Media



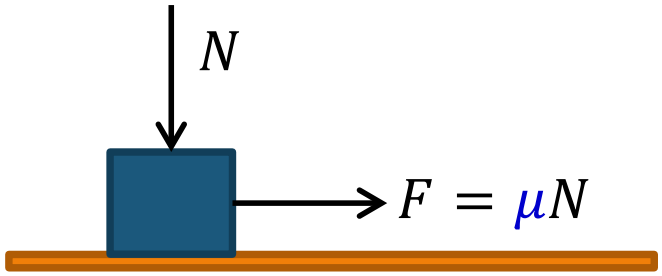
Wet nonBrownian Suspensions



What happens for Brownian suspensions?



Analogy with classical mechanics:



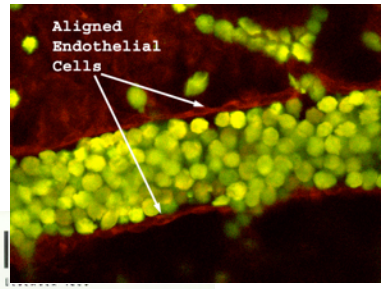
NonBrownian suspensions

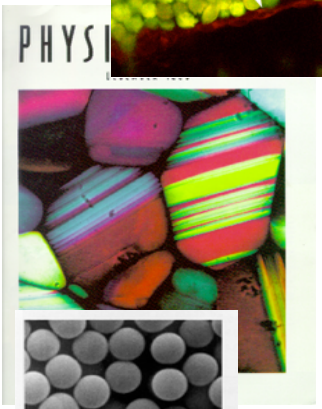
$$\sigma = \eta\dot{\gamma}\,\eta_s(\phi)$$
$$\Pi = \eta\dot{\gamma}\,p(\phi)$$
$$\mu = \frac{\sigma}{\Pi} = \frac{\eta_s(\phi)}{p(\phi)}$$
$$I_v = \eta\dot{\gamma}/\Pi = 1/p(\phi)$$

Brownian suspensions

$$\sigma = \eta\dot{\gamma}\,\eta_s(\phi, Pe_\sigma) + \sigma^{yield}(\phi)$$
$$\Pi = nkT\left(\hat{\Pi}^{eq}(\phi) + \Delta\hat{\Pi}(\phi, Pe_\sigma)\right)$$
$$\mu = \frac{\sigma}{\Pi} = \frac{\eta_s(\phi) Pe_{\dot{\gamma}}}{\hat{\Pi}^{eq}(\phi) + \Delta\hat{\Pi}(\phi, Pe_\sigma)}$$
$$I_v \quad , \quad Pe_\sigma = \sigma a^2/\eta D$$

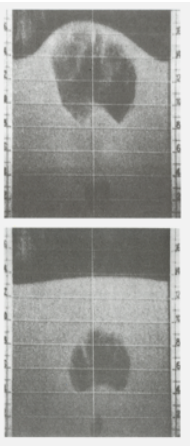
Two-phase particulate matter





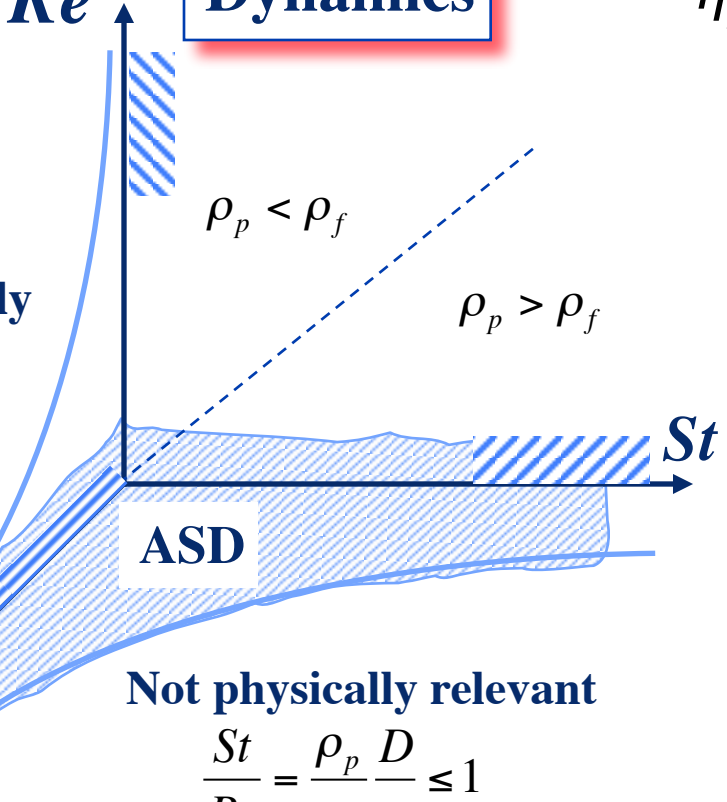
Bubble Dynamics

$$Re = \frac{\rho_f a^2 \dot{\gamma}}{\eta_f}$$



$$St = \frac{\rho_p a^2 \dot{\gamma}}{\eta_f}$$

Granular Dynamics



Not physically relevant

$$\frac{Re}{Pe} = \frac{D}{\nu} << 1$$

Not physically relevant

$$\frac{St}{Pe} = \frac{\rho_p}{\rho_f}\frac{D}{\nu} \leq 1$$

$$Pe = \frac{a^2 \dot{\gamma}}{D}$$

Stokesian Dynamics

The End